AP Calculus AB Summer Assignment

Welcome to AP Calculus AB!

This assignment is designed to help you prepare for your upcoming AP Calculus AB class. All of the topics explored in this assignment should be a review of material you learned in your pre-calculus course; however, there may be a few topics that you may not have spent much time practicing in your prior math classes (for example, simplifying rational expressions).

I strongly encourage you try your best to work through this assignment: take note of the questions you may have had a difficult time completing and return to school ready to review! We will spend the first week of school reviewing this content, but we will not have enough time to do ALL of the questions so please come prepared with SPECIFIC questions.

I'm looking forward to exploring AP Calculus AB with you! This course can be challenging, but I have full confidence in you: YOU CAN DO IT! ☺

There will be a test over the content explored in this packet
THE FIRST WEEK OF SCHOOL!

If you have any questions or concerns, please feel free to contact at grameka@lc-ps.org

Good luck! ☺
Mrs. Gramer

Prerequisites for Calculus AB:

The following is a list of concepts that you have learned in one (or more) of your math classes throughout your school career. The summer assignment will not review ALL of these topics, but you will be expected to know ALL of them. Please be sure to review and practice these topics until you are confidently comfortable with all of them. You are permitted to use any resources you wish – previous class notes, online resources, textbooks, etc. – ***but please be aware that you are NOT to use a calculator to complete this summer assignment.***

Prior Mathematical Knowledge:

Upon entering AP Calculus AB, you should be able to…

- Calculate the slope of a function (positive, negative, zero, and undefined)
- Write a linear equation in slope-intercept form, standard form, and point-slope-form
- Identify x-intercepts, y-intercepts, and points of intersection
- Identify graphs of "parent" functions and describe transformations of the parent functions
- Identify the domain and range of a given function
- Graph and write piecewise functions on a given domain
- Apply function notation to function operations and compositions
- Identify whether a function is even/odd or has an axis of symmetry
- Apply exponent rules and be able to convert between exponential and rational form
- Identify if a relation (and it's inverse) is a function (vertical line test)
- Identify if a function is "one-to-one" (horizontal line test)
- Find inverse functions and describe their domain and range
- Solve all types of equations (linear, quadratic, polynomial, rational, radical, exponential, logarithmic)
- Evaluate the six basic trig functions (and their reciprocals) around the unit circle
- Graph the six basic trig functions (and their reciprocals)
- Identify trigonometric identities (ratio, reciprocal, Pythagorean, and double-angle – memorize these!)

Factoring Rules:

1. **Find the GCF** – something that ALL the terms can be divided by.
   a. *Example: The GCF of* $12x^3 - 6x^2 + 18x = 3x(4x^2 - 2x + 6)$

2. **Check for "Perfect Squares"!** – A & C positions only!
   a. If there are 3 terms, both sets should have the sign of the middle term.
      i. *Example:* $9x^2 - 24x + 16 = (3x - 4)(3x - 4)$ *or* $(3x - 4)^2$
      ii. *Example:* $4x^2 + 20x + 25 = (2x + 5)(2x + 5)$ *or* $(2x + 5)^2$

   b. If there are 2 terms, one set is positive and one set is negative.
      i. *Example:* $25x^2 - 49 = 25x^2 + 0x - 49 = (5x + 7)(5x - 7)$

3. **AC Method** – sing your factoring song!

Factoring Song for the AC METHOD:

Steps to Perform the AC Method:	**Example:**
(To the tune of Twinkle, Twinkle, Little Star)	$6x^2 - 11x - 10$
Drop A, Drop C, Multiply!	$(6x^2 \quad)(\quad -10)$
Find the factors that add to B.	**AC:** $\mathbf{6x^2 * -10 = -60x^2}$ 1 and 60 2 and 30 3 and 20 **4 and 15 → $4x - 15x = -11x$** 5 and 12 6 and 10
GCF and GCF	$(6x^2 + 4x)(-15x - 10)$ $2x(3x + 2) - 5(3x + 2)$
Write the match, and combine the rest.	$(3x + 2)(2x - 5)$
Now I know how to FACTOR! Never forget our Factoring Rules! ☺	

Trigonometric Identities: ***Please have these memorized along with*** $\boldsymbol{sin(2x) = 2sinxcosx.}$

Reciprocal Identities . . .

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

Quotient Identities . . .

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities . . .

$$\sin^2 x + \cos^2 x = 1 \qquad 1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Finding Intercepts & Points of Intersection

Find the x and y-intercepts for each of the following equations.

1. $y = 2x - 3$	2. $y = x^2 + x - 2$
3. $y = x^2\sqrt{9 - x^2}$	4. $y = \frac{x-1}{x-2}$

Find the point(s) of intersection of the following systems of equations.

5. $\begin{cases} x + y = 2 \\ 2x - y = 1 \end{cases}$	6. $\begin{cases} x + y = 7 \\ 3x - 2y = 11 \end{cases}$
7. $\begin{cases} x^2 + y^2 = 5 \\ x - y = 1 \end{cases}$	8. $\begin{cases} y = x^3 \\ y = x \end{cases}$

Function Notation & Composed Functions

Evaluate the following expressions.

Given $f(x) = 2x - 3$ and $g(x) = \sqrt{x + 3}$ find the following:			
1. $f(0)$	2. $g(-2)$	3. $f(-3)$	4. $g(6)$
5. $f(b)$	6. $g(c)$	7. $f(2 + h)$	8. $g(x + h)$

Given $f(x) = x^2 + 3x - 10$, $g(x) = 3x^2 - 1$, and $h(x) = x + 5$, find each of the following function operations. State the domain with any restrictions.

9. $(f - g)(x)$	10. $(h * g)(x)$	11. $\left(\frac{h}{f}\right)(x)$
12. $(g - h)(x)$	13. $(g + h)(x)$	14. $\left(\frac{f}{h}\right)(x)$

Given $f(x) = 3x - 1$ and $g(x) = \frac{1}{x-3}$ find the following:

15. $\frac{f(x)-f(1)}{x-1}$	16. $\frac{g(x)-g(5)}{x-5}$

Find the composed function $h(x)$, where $h(x) = f(g(x))$.

17. $f(x) = sinx$ and $g(x) = 3x + 1$	18. $f(x) = \sqrt{x}$ and $g(x) = cosx$
19. $f(x) = cosx$ and $g(x) = sinx$	20. $f(x) = \sqrt{x}$ and $g(x) = 1 + x^2$

Identify the two functions $f(x)$ and $g(x)$ that create the composed function $h(x) = f(g(x))$.

21. $h(x) = (2x - 5)^4$	22. $h(x) = sec^5 x$	23. $h(x) = \csc(x^4)$

Proving Identities Using Trigonometric Functions

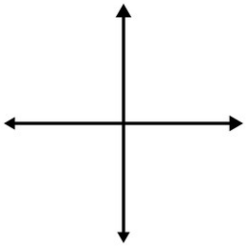
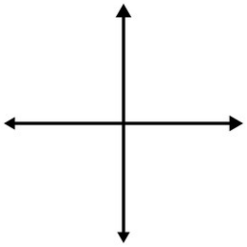
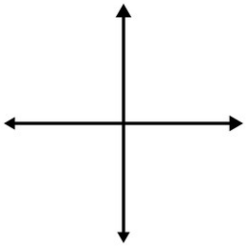
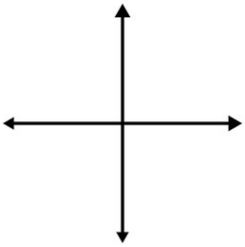
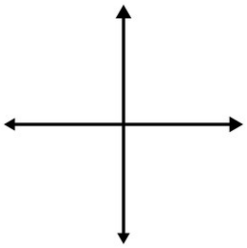
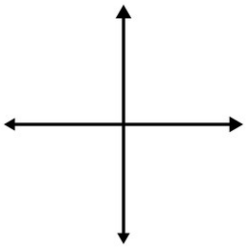
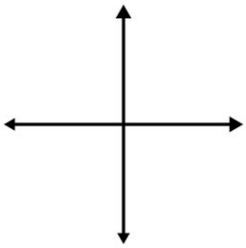
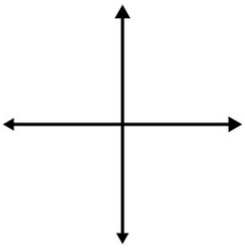
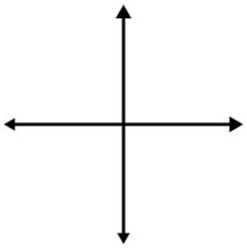
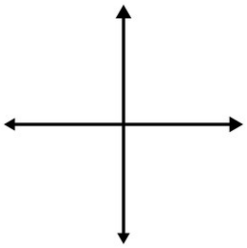
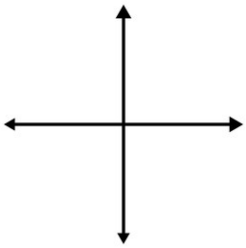
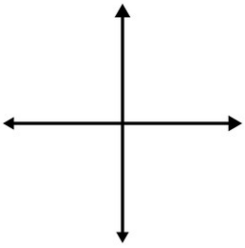
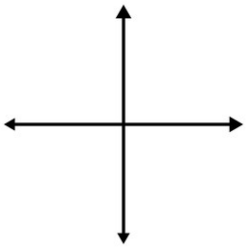
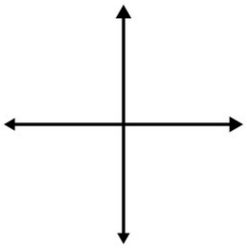
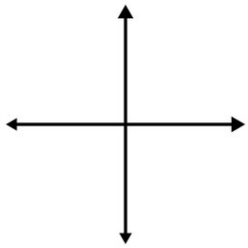
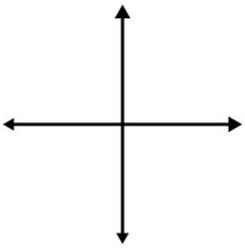
You need to be able to recognize situations where substitutions with the basic trigonometric identities would be appropriate. Remember that you can only make substitutions on ONE side of the equal sign, then try to continue to make substitutions and simplify until both sides are equal. ***(You may need to look up some of these trig identities online!)***

1. $\frac{csc^2x-1}{cot^2x} = 1$	2. $cos^2x - sin^2x = 1 - 2sin^2x$
3. $(1 + tan^2x)(1 - sin^2x) = 1$	4. $cos^2x - sin^2x = 2cos^2x - 1$
5. $sec^2x = \frac{sin^2x+cos^2x}{cos^2x}$	6. $csc^2xtan^2x - 1 = tan^2x$
7. $\frac{1}{cot^2x+1} = 1 - cos^2x$	8. $sin2x = \frac{2tanx}{1+tan^2x}$
9. $\frac{sec^2x}{sec^2x-1} = csc^2x$	10. $\frac{sin2x}{2sinx} = cosx$

Graphs of Parent Functions

"Parent Functions" are basic functions that you should be able to RECOGNIZE in equation form, and VISUALIZE as a graph. You should also be aware of how these parent functions can be translated/shifted around the coordinate plane. Sketch each of the functions below and identify their domains using appropriate interval notation. (If you don't know what the graph looks like, use your calculator or look it up online.)

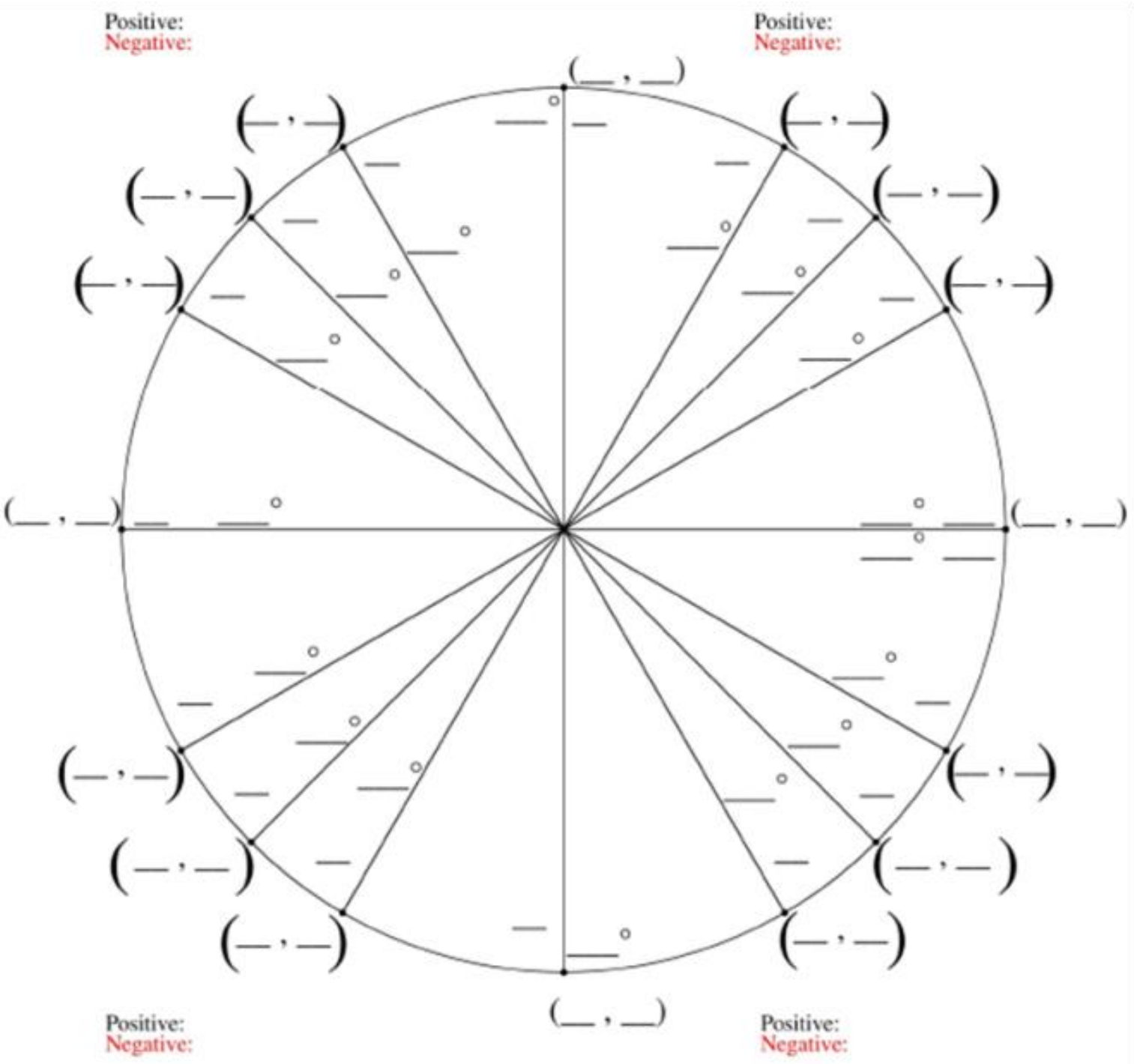
You will need to have this memorized!!! ☺

1. $y = c$ and $x = c$  Domain:	2. $y = x$  Domain:	3. $y = x^2$  Domain:	4. $y = x^3$  Domain:
5. $y = \lvert x \rvert$  Domain:	6. $y = \sqrt{x}$  Domain:	7. $y = \sqrt[3]{x}$  Domain:	8. $y = x^{2/3}$  Domain:
9. $y = \frac{1}{x}$  Domain:	10. $y = \frac{1}{x^2}$  Domain:	11. $y = \sqrt{1 - x^2}$  Domain:	12. $y = e^x$  Domain:
13. $y = \ln x$  Domain:	14. $y = sinx$  Domain:	15. $y = cosx$  Domain:	16. $y = tanx$  Domain:

The Unit Circle

Use this unit circle to help you visualize the angles and recall the values of the six trigonometric functions for standard angles measures on the unit circle. Remember the x-value identifies the exact cosine value and the y-value identifies the exact sine value of the given angle.

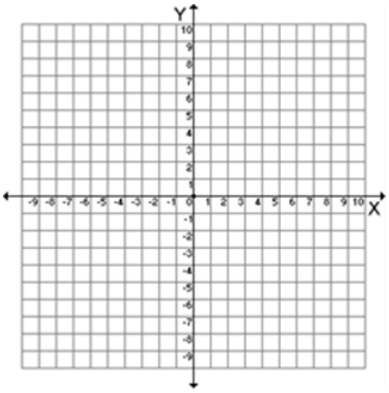
You will need to have this memorized!!! ☺



Analyzing Rational Functions

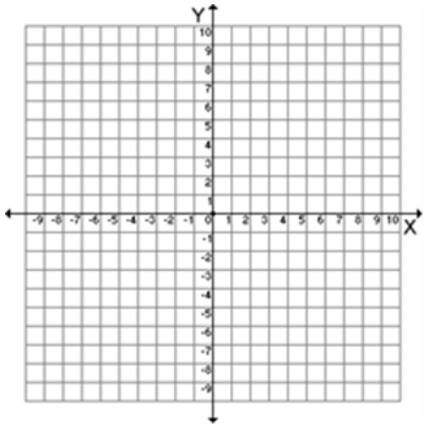
For each of the following rational functions, identify the key elements and sketch a graph.

1. $y = \frac{3x-6}{x^2-4x-5}$



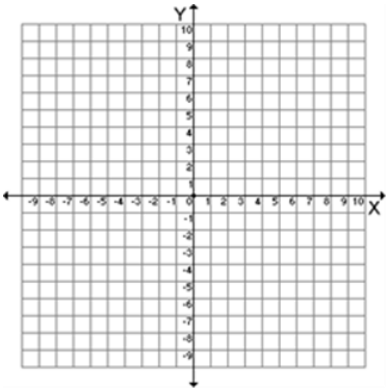
X-int: _____
VA: _____
HA: _____
Holes: _____
Domain: _____
Range: _____

2. $y = \frac{5x^3+15x^2}{x^2-9}$



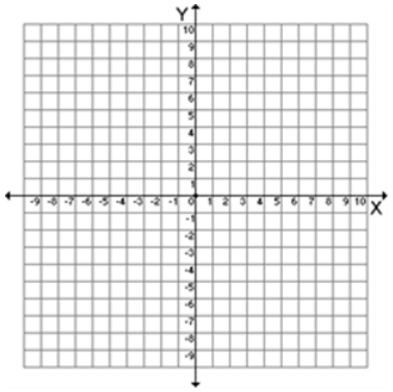
X-int: _____
VA: _____
HA: _____
Holes: _____
Domain: _____
Range: _____

3. $y = \frac{3x^2-6x}{x^2-4x+4}$



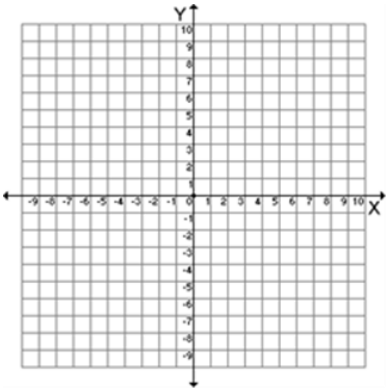
X-int: _____
VA: _____
HA: _____
Holes: _____
Domain: _____
Range: _____

4. $y = \frac{x^3-8}{x-2}$



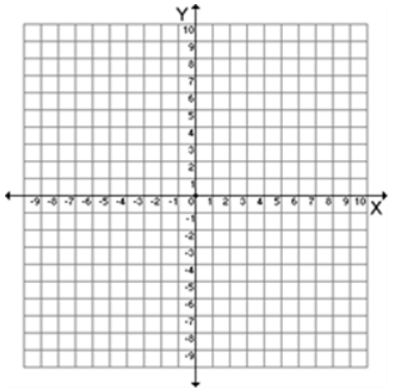
X-int: _____
VA: _____
HA: _____
Holes: _____
Domain: _____
Range: _____

5. $y = \frac{4x}{4x^2-24x}$



X-int: _____
VA: _____
HA: _____
Holes: _____
Domain: _____
Range: _____

6. $y = \frac{6x-5}{4x-12}$



X-int: _____
VA: _____
HA: _____
Holes: _____
Domain: _____
Range: _____

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Simplifying Expressions

Simplify the following expression COMPLETELY! This means you final answer should have no complex fractions, no negative exponents, no radicals in the denominator, and all terms should be written in simplest form (i.e. combine like terms and single logarithmic expressions).

1. $x\left(3x+2\left(x-(2x+1)\right)\right)$	2. $\frac{\sqrt{x}}{\sqrt{x}+5}$	3. $\frac{9x^2-3x-2}{9x^2-4} * \frac{3x^2-10x-8}{27x^3+1}$
4. $\frac{x^2+5x-24}{2x+2} \div \frac{3x+24}{x^2-8x-9}$	5. $\left(\sqrt{3}-\sqrt{7}\right)\left(2\sqrt{7}+5\sqrt{3}\right)$	6. $\frac{1}{x+3} - \frac{4}{x^2+4x+3}$
7. $\frac{5-3\sqrt{7}}{9\sqrt{3}}$	8. $\frac{\frac{x+y}{2x-y}}{\frac{x+y}{-2x+y}}$	9. $\frac{\frac{1}{x}-\frac{1}{y}}{1+\frac{1}{x}}$
10. $\frac{2}{4x+12}+\frac{7}{x+3}$	11. $\frac{\sqrt{5}+3}{-\sqrt{7}-1}$	12. $\frac{3+\frac{5}{x+2}}{3-\frac{10}{x+7}}$
13. $3\log_2 x + 5\log_2 y$	14. $9\ln y - 8\ln x$	15. $\frac{(x+y)\left(\frac{1}{x}-\frac{1}{y}\right)}{(x-y)\left(\frac{1}{x}+\frac{1}{y}\right)}$
16. $\frac{1}{3}x^{-\frac{2}{3}} - \frac{7}{3}x^{\frac{7}{3}}$	17. $\frac{3}{2}x^{\frac{1}{2}} + 2x^{-\frac{3}{2}}$	18. $\frac{(-3x+1)\left(\frac{1}{2}\right)\left(x^{-\frac{1}{2}}\right)+\left(x^{\frac{1}{2}}\right)(3)}{(3x+1)^2}$

AP Calculus AB Summer Assignment

Solving Equations

Solve the following equations. ***You may need to factor some of these!***

1. $\frac{3}{4}-\frac{2x}{4x-24}=\frac{8}{x-6}$	2. $\frac{x+10}{x-7}=\frac{8}{9}$	3. $\frac{9}{x-7}=\frac{6}{x}$
4. $\sqrt{x-3}=x-5$	5. $\sqrt[3]{3x+4}=-5$	6. $-5\sqrt[5]{3x+2}=-10$
7. $5m^2+40m-100=0$	8. $4x^2+9x=-2$	9. $2x^2=-5x-2$
10. $4x^2-81=0$	11. $0=75x^2-48$	12. $5x^2-14x-3=0$
13. $3x^3-5x^2=21x-35$	14. $x^4+12x^2-64=0$	15. $3x^3=24x$

16. $tan^2xcosx=tan^2x$ on $[0,2\pi]$	17. $\sqrt{3}tanx+1=0$ on $[0,2\pi]$
18. $4sin^2x-3=0$ on $[0,2\pi]$	19. $2sin^2x-sinx-1=0$ on $[0,2\pi]$